

# Related Works

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## Abstract

Special Relativity carries an assumption: the observer is a point. Einstein knew it was important. He recorded the debt twice: borrowed rods and clocks, to be eliminated "*at a later stage of the theory.*" And the founders of Quantum Mechanics did the same.

This record documents the century of that debt. The theorems that barred the finite body and were read as final. The axiomatic tradition that pared those theories' assumptions to almost nothing and never put this one on the list. The paradoxes that returned every decade, each filed as a puzzle.

This record then documents the mathematics. Folds, caustics, catastrophes that arrived from another direction, carrying every tool the payment needed. This became the later stage.

The claim is narrow: the assumption was identified but ultimately undeclared, the replacement became available decades ago, at which time no new mathematics was required. This is a historical record of those works. Comments appear in the margins relating the same questions and works to The Observer Lemma. The consequences are derived there.

# Related Works

## I. An Assumption Not Declared

*"Concepts which have proved useful for ordering things easily assume so great an authority over us, that we forget their terrestrial origin and accept them as unalterable facts."*

— Einstein, obituary for Ernst Mach, 1916

Assumptions that "appear so obvious that people do not know what they are assuming," Whitehead saw,<sup>†</sup> are not hidden, just never written down. The observer of special relativity is  $x(\tau)$ : a worldline, a point carrying a clock, an assumption of the kind Mashhoon wrote was "simply adopted by Einstein for rods and clocks," and Reichenbach stipulated is notation, not an axiom: "neither true nor false." And as Whitehead wrote, "Civilization advances by extending the number of important operations which we can perform **without thinking about them.**"<sup>‡</sup>

Einstein stated two assumptions: 1. The laws of physics are the same for every observer in uniform motion; 2. No matter how its source moves, each measures light passing at the same speed. Over a century: the second proved unnecessary; the first was rebuilt from causal order alone.<sup>1\*</sup> The idealized point observer, the useful fiction (nützliche Fiktion) that the **observer has no size, was not on the list.**<sup>2‡</sup> This record puts it there.

Einstein raised the issue in 1921, addressing the Prussian Academy:

*"The idea of the measuring-rod and the idea of the clock co-ordinated with it in the theory of relativity do not find their exact correspondence in the real world. It is also clear that the solid body and the clock do not in the conceptual edifice of physics play the part of irreducible elements, but that of composite structures, which may not play any independent part in theoretical physics."*<sup>3‡</sup>

28 years later, in the Autobiographical Notes, he said it again. There he wrote down the debt:

*"One is struck [by the fact] that the theory... introduces two kinds of physical things, i.e., (1) measuring rods and clocks, (2) all other things, e.g., the electromagnetic field... strictly speaking measuring rods and clocks would have to be represented as solutions of the basic equations... it was better to permit such inconsistency, with the **obligation, however, of eliminating it at a later stage of the theory.**"*<sup>4§</sup>

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\* Two audits, one note. The second assumption shown unnecessary: W. von Ignatowski (1910), tightened by P. Frank and H. Rothe (1911). The first rebuilt from causal order: A. A. Robb (1914), A. D. Alexandrov (1950), E. C. Zeeman (1964). Full citations where their stories are told, § IV.

† H. Vaihinger, *Die Philosophie des Als Ob* (Berlin: Reuther & Reichard, 1911). The era possessed a technical name for assumptions known to be false and kept for their use. The point observer was never granted even the name. A. N. Whitehead, *Science and the Modern World* (1925): such assumptions persist because "no other way of putting things has ever occurred to them"; adherents "unconsciously presuppose" them — "not hidden" translates the witnesses. B. Mashhoon, "Nonlocal Special Relativity" (2008): the "hypothesis of locality," exactly valid only for "pointlike coincidences"; Mashhoon audited the observer's instant, this record its extent. H. Reichenbach, *The Philosophy of Space and Time* (1928): coordinative definitions. Closing line: Whitehead, *An Introduction to Mathematics* (1911), the chapter on symbolism; emphasis added.

‡ A. Einstein, *Geometrie und Erfahrung* (Berlin: Springer, 1921); address to the Prussian Academy of Sciences, 27 January 1921. English translation in *Sidelights on Relativity* (London: Methuen, 1922); quoted wording verified against the Methuen text. In the same address Einstein reasons throughout with the "practically-rigid body," an object he concedes does not exist in his own theory. The deferral is explicit there too: these ideas "must still be employed as independent ideas; for we are still far from possessing such certain knowledge of theoretical principles as to be able to give exact theoretical constructions of solid bodies and clocks."

§ A. Einstein, "Autobiographical Notes," in P. A. Schilpp, ed., *Albert Einstein: Philosopher-Scientist* (Evanston: Library of Living Philosophers, 1949), pp. 59–61. The passage is usually quoted in fragments. The closing clause, the obligation, is the part most often omitted, and the part this record is about. In the original: "es war besser, solche Inkonzsequenz zuzulassen — mit der Verpflichtung jedoch, sie in einem späteren Stadium der Theorie zu eliminieren."

With the obligation of eliminating it at a later stage, the founder of General Relativity twice called his observer a borrowed instrument. Twice he recorded the debt.‡§

What follows is an account of that unpaid debt. The men who tried to pay it and were stopped by their own theorems. The men who shaved the theory's assumptions down to almost nothing and never touched this one. And the mathematicians who for entirely different reasons built every tool the payment needed.

I came to this story from outside, over forty years ago, and I appear in it only in the margins. My own dead-ends landed on famous ones. It was the **same question** standing in a blind spot. Shared by everyone who inherited the notation, me included, for decades.

## II. The Instruments Borrowed, 1905–1913

*"The concept of length is therefore fixed when the operations by which length is measured are fixed."*  
— Bridgman, 1927

**1905 — Einstein.** The founding paper builds its kinematics out of hardware. "*Zur Elektrodynamik bewegter Körper*" does not begin with spacetime; there was no spacetime yet. It begins with instruments and a procedure:

*"Let there be given a stationary rigid rod..."*<sup>5\*</sup>

*and a definition of time resting on the observation that*

*"all our judgments in which time plays a part are always judgments of simultaneous events."*<sup>\*</sup>

Time established "by definition," Einstein wrote: light bounced between clocks at rest. Clocks that never get, Brown and Pooley wrote, "a dynamical treatment of the forces holding together the constitutive parts of moving rods and clocks." "It was better," Einstein said of it later. Also, by his own accounting, a loan.<sup>\*</sup>

**1906 — Planck.** The first to countersign the loan. Within a year Planck had produced the first relativistic dynamics and assigned the first doctoral work on the theory. Then he did something that deserves more remembrance than it gets. Kaufmann's electron-deflection data appeared to refute the theory.<sup>6†</sup> Planck examined the experiment instead of the theory, judged the data too insecure to overturn so coherent a structure, and was vindicated when the measurements were redone. Keep that lesson; this record needs it again before the end. An experiment unseats a theory, but only an experiment that survives the same scrutiny the theory has.

**1908 — Minkowski.** Then the loan was poured into bronze. The Cologne address gave physics its most quoted overture: "*Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality.*"<sup>7‡</sup> Less quoted, and

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\* A. Einstein, "Zur Elektrodynamik bewegter Körper," *Annalen der Physik* 17 (1905): 891–921, § 1; both quoted phrases from the kinematical part. English translation by W. Perrett and G. B. Jeffery in *The Principle of Relativity* (London: Methuen, 1923). In the original: "Sei ein ruhender starrer Stab gegeben; derselbe besitze, mit einem ebenfalls ruhenden Maßstabe gemessen, die Länge  $l$ ." And: "daß alle unsere Urteile, in welchen die Zeit eine Rolle spielt, immer Urteile über gleichzeitige Ereignisse sind. The synchronization is definitional in the original: 'wir setzen durch Definition fest, daß die 'Zeit'...' — 'we establish by definition that the "time" required by light to travel from A to B equals the "time" it requires to travel from B to A.

† M. Planck, "Das Prinzip der Relativität und die Grundgleichungen der Mechanik," *Verhandlungen der Deutschen Physikalischen Gesellschaft* 8 (1906): 136–141; the re-analysis: "Die Kaufmannschen Messungen der Ablenkbarkeit der  $\beta$ -Strahlen in ihrer Bedeutung für die Dynamik der Elektronen," *Physikalische Zeitschrift* 7 (1906): 753–761. Kaufmann's deflection measurements, published the same year, favored the rival electron theories of Abraham and Bucherer; Planck's re-analysis found the data insufficient to decide, and later measurements (Bucherer 1908 and after) reversed the verdict. The episode is the earliest instance of a pattern this record returns to in § V: structure, coherently held, may outlast an experiment, but only until a better experiment. Planck knew both sides of this coin. Forty-two years later, in the *Wissenschaftliche Selbstbiographie*: "Eine neue wissenschaftliche Wahrheit pflegt sich nicht in der Weise durchzusetzen, daß ihre Gegner überzeugt werden und sich als belehrt erklären, sondern vielmehr dadurch, daß ihre Gegner allmählich aussterben." A new truth does not win by convincing its opponents; its opponents eventually die.

‡ H. Minkowski, "Raum und Zeit," *Physikalische Zeitschrift* 10 (1909): 104–111; address delivered at the 80th Assembly of German Natural Scientists and Physicians, Cologne, 21 September 1908. In the original: "Von Stund' an sollen Raum für sich und Zeit für sich völlig zu Schatten herabsinken, und nur noch eine Art Union der beiden soll Selbständigkeit bewahren." Translation: Perrett and Jeffery, *The Principle of Relativity*.

more consequential for this account, it gave the observer its permanent body: "The whole universe is seen to resolve itself into similar world-lines, and... physical laws might find their most perfect expression as reciprocal relations between these world-lines."<sup>8§</sup>

A worldline has no thickness. From September 1908 forward the observer was a curve; Einstein sealed it, resolving his own hole argument: all verification is "nothing but" the meetings of material points; coincidences of worldlines.<sup>||</sup> The physical content of the theory, Norton writes, is "exhausted" by their intersections.

The dates of my work enter here, out of order. Mine will jump; theirs will not. In the late 1980s I asked what gravity looks like for a mass if the mass is not a point on one of Minkowski's world-lines but a path: a closed orbit in curved space, circling at the speed of light, hiding the missing space. Causing the curve, not just representing it. But the path's head did not quite meet its tail, so there was a gap, and the gap had a direction. That pinwheel I saw instantly: electromagnetism, from the same geometry as gravity.

**1912 — Grossmann.** The second loan. When the geometry of curvature outran him, Einstein went to his friend, the mathematician Marcel Grossmann: "Grossmann, you must help me, or I'll go crazy!"

Grossmann went to the literature and came back with the problem already solved: Riemann, Ricci, Levi-Civita. Einstein's thanks, in print, is exact about what the help was: it "spared me the study of the relevant mathematical literature."<sup>¶</sup> The calculus worked. He ran it on points and worldlines, not shapes. It gave him the famous relation. It also gave him singularities, and he spent the rest of his life trying to outlaw what the borrowed instrument produced:

*"For a singularity brings so much arbitrariness into the theory that it actually nullifies its laws. Every field theory, in our opinion, must therefore adhere to the fundamental principle that singularities of the field are to be excluded."*

The principle was his. The instrument could not meet it. Both loans, the hardware of 1905 and the geometry of 1912, carried the same unexamined passenger.

I took the loan myself. In the mid 1990s Minkowski's forms were what I had, and in the linear regime they delivered: the circling paths produced gravity, resolving exactly to Einstein's weak-field equations.<sup>10¶</sup> A restatement, at best. I knew it. The forms carried me exactly as far as they carry everyone, and no farther. A worldline cannot compute a surface. The missing space needed that surface, so stayed beyond my reach; the mathematics that could compute it was already written, but I had not found it. The promise was what kept me going. What I found those years: the same equations, from the opposite assumptions.

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§ Minkowski, "Raum und Zeit." The term is Minkowski's own: Weltlinie. In the original: "Die ganze Welt erscheint aufgelöst in solche Weltlinien, und ... die physikalischen Gesetze dürften ihren vollkommensten Ausdruck als Wechselbeziehungen zwischen diesen Weltlinien finden." It is worth pausing on how total the victory of this coinage was; within a generation "observer" and "worldline" had become interchangeable in calculation, and the substitution no longer registered as one. The sealing is Einstein's point-coincidence argument, resolving his "hole" argument in the Grundlage (cited at ||): all verification "nothing but" such meetings of material points. J. D. Norton, "The Hole Argument," Stanford Encyclopedia of Philosophy: a theory's content is "exhausted by the catalog of the spacetime coincidences it licenses."

|| Three sources share this note. The plea is recollection: L. Kollros, "Albert Einstein en Suisse: Souvenirs," Helvetica Physica Acta, Suppl. 4 (1956): 271; in the original: "Grossmann, Du musst mir helfen, sonst werd' ich verrückt!" The printed thanks: A. Einstein, "Die Grundlage der allgemeinen Relativitätstheorie," Annalen der Physik 49 (1916): 769–822, introductory acknowledgment. The displayed passage: A. Einstein and N. Rosen, "The Particle Problem in the General Theory of Relativity," Physical Review 48 (1935): 73–77. His last published word on the subject is of a piece: the 1939 investigation concluding that "Schwarzschild singularities" do not exist in physical reality: Annals of Mathematics 40 (1939): 922–936.

¶ A. Einstein, "Näherungsweise Integration der Feldgleichungen der Gravitation," Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften (1916): 688–696. Keep the disturbances only to first order; what remains is a wave equation, with waves traveling at  $c$ : these are the linear forms meant here. Näherungsweise: "by approximation." His own title for them. The result was old; only the route was new. The claim in this record is agreement with these linear forms in the far-field regime, and nothing more. And one citation I cannot pay properly: sometime in the 1990s I saw another derivation of the same kind, from Scandinavia I believe, a massless-particle theory in which two massless particles orbit each other at the speed of light, the same mathematics for gravity in the far-field regime. I have searched for years and cannot find it. If a reader can place it, the record wants the citation.

### III. The Finite Body Rebuffed, 1909–1921

*"Uniform rotation of rigid bodies and the theory of relativity... leads to a contradiction."*  
— Ehrenfest, 1909

In less than a year someone asked what happens when the observer has a size:

**1909 — Born.** Born asked first, for a serious physical reason: he wanted the electron to be a real extended object, so he needed to know what "rigid" can mean when no part of a body may outrun light.<sup>11\*</sup> His definition was careful and covariant: each material element keeps a constant proper distance from its neighbors; rigidity belongs to the motion, not the substance. It was immediately catastrophic.

**1909 — Ehrenfest.** The wound was published within months, in under a page:

"Let a relatively rigid cylinder of radius  $R$  and height  $H$  be given. A rotation about its axis of symmetry... is finally imparted to it. [Its] periphery... must show contraction...  $2\pi R' < 2\pi R$ ... [But] the radius  $R$  cannot show a contraction."<sup>12†</sup>

The "body" was the trap. The body was the trapped space I saw around 1992.

**1910 — Herglotz; Noether.** Two theorems closed the case. Born-rigid motions form a three-parameter family; a classical rigid body has six. A Born-rigid body cannot even be spun up from rest.<sup>13‡</sup>

**1911 — Laue.** The epitaph: any extended body in special relativity must possess infinitely many degrees of freedom, since an impulse applied at one point cannot influence the far side faster than light.<sup>14§</sup>

**1921 — Pauli.** In the great Encyklopädie article he filed the affair as settled doctrine: "If thus the concept of a rigid body has no place in relativistic mechanics, it is nevertheless useful and natural to introduce the concept of a rigid motion of a body."<sup>15¶</sup> Rigidity belongs to motions, not bodies.

Five first-rate physicists examined the finite body at the foundations of relativity. Every theorem they proved was correct. But the theorems said something narrower. Pauli himself, in the same article: the contraction of a rod is "not an elementary but a very complicated process," resting on laws "as yet unknown to us, which determine the cohesion of the electron itself."<sup>¶</sup> An extended body cannot be rigid. It cannot be a scaled-up point. The answer I saw was in Pauli's rebuttal: rigidity belongs to motions, not bodies. Non-rigidity is information: the way an extended structure transforms says something about where transformations come from. A reading no one had cause to attempt. It became the heart of my work. Born\* moved on, to a different revolution. The point observer won.

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\* M. Born, "Die Theorie des starren Elektrons in der Kinematik des Relativitätsprinzips," *Annalen der Physik* 335, no. 11 (1909): 1–56. The dual volume numbering (335/30) is original to the *Annalen* and a standing trap for citation checkers. The different revolution: the probability interpretation, M. Born, "Zur Quantenmechanik der Stoßvorgänge" (1926), cited again at § V's dice note; Nobel Prize, 1954.

† Ehrenfest, "Gleichförmige Rotation starrer Körper und Relativitätstheorie," *Physikalische Zeitschrift* 10 (1909): 918. The displayed quotation is a condensation of Ehrenfest's single paragraph. In the original: "Ein relativ-starrer Zylinder vom Radius  $R$  und der Höhe  $H$  werde gegeben... [seine Peripherie] muß eine Kontraktion zeigen... der Radius  $R$  dagegen kann keine Kontraktion zeigen."

‡ G. Herglotz, "Über den vom Standpunkt des Relativitätsprinzips aus als 'starr' zu bezeichnenden Körper," *Annalen der Physik* 336 (1910): 393–415; F. Noether, "Zur Kinematik des starren Körpers in der Relativtheorie," *Annalen der Physik* 336 (1910): 919–944.

§ M. von Laue, "Zur Diskussion über den starren Körper in der Relativitätstheorie," *Physikalische Zeitschrift* 12 (1911): 85–87.

¶ W. Pauli, "Relativitätstheorie," in *Encyklopädie der mathematischen Wissenschaften*, vol. V/2 (Leipzig: Teubner, 1921); English translation, *Theory of Relativity* (London: Pergamon, 1958). Pauli was twenty-one when it appeared; Einstein reviewed it with open admiration. In the original: "Wenn somit der Begriff des starren Körpers in der relativistischen Mechanik keinen Platz hat, so ist es doch zweckmäßig und naturgemäß, den Begriff der starren Bewegung eines Körpers einzuführen." The open half, same article: the atomistic explanation is not to be abandoned; p. 15 of the translation.

My object was the photon and how its motion could create the electron. But I, like everyone, inherited that defeat of famous pedigree. I did not set out to correct Einstein; I believed him right at every scale. His equations show how mass curves, but never why. Where is the missing space was my whole question. Years later I replaced mass with the space the photon hides, and the rigid body with the shape its motion forms. Then I did the math. Far field: exact agreement. Compton scale: his forms break, as he knew; missing-space forms continue. I hadn't gone looking for that. The why demoted the how: his equations are the far-field approximation. His singularity lament answered. The missing space obeys his principle unasked; the point never could, even when begged.

## IV. The Axiomatic Retreat, 1910–1984

"*Causality implies the Lorentz group.*" — Zeeman, 1964 (title)

While the finite body was being exiled, a quieter tradition asked how little the Lorentz transformation actually requires. Its century of silence on the point observer is the sharpest evidence this record has.

**1910 — von Ignatowski.** The program opens with a result that still surprises students: Einstein's second assumption is unnecessary. Linearity, reciprocity, isotropy, and the group property are enough. The transformation follows with one constant left over, an invariant speed the axioms cannot fix and cannot avoid.<sup>16\*</sup> Frank and Rothe tightened the derivation within a year. The assumption everyone had argued about turned out to be a calibration, not a foundation. The quiet assumptions were doing the real work. The tradition promptly began paring those too.

**1914 — Robb.** At Cambridge, at a self-chosen distance from the continental debates, Robb rebuilt the whole theory from a single primitive relation: after.<sup>17†</sup> No rods, no clocks, no light assumption. Causal order alone, carrying the full geometry, in a book of austere architecture that physics largely declined to read.

**1924 — Reichenbach.** The theory's empirical content axiomatized; the conventionality of simultaneity exposed.<sup>18‡</sup>

Look at what this tradition achieved, and what it never touched. Light: eliminated. Clocks and rods: eliminated, by Robb, in 1914. Simultaneity: a convention. Metric structure: from causal order alone, up to scale. The observer stayed what Minkowski had made it; in Zeeman's theorem there is no observer at all, only the points of  $\mathbb{R}^4$ . This is not their failure. An axiomatist shortens the list in front of them, and the point observer was not on the list. Einstein twice recorded that debt. The debt was rational: the mathematics worked, and nothing existed to replace it. "A later stage," he wrote. It never arrived. So the observer having size may look like an addition. In proper order the point is the bigger assumption: exact location, zero extent, free of charge, at every event, forever. Structure is not added. It is just not removed.

**1950; 1964 — Alexandrov; Zeeman.** Here the causal program reached its summit, twice, independently: the causal automorphisms of Minkowski space are exactly the Lorentz group with translations and dilations.<sup>‡</sup> Zeeman published the theorem under the most economical title in the literature, given above as this section's epigraph.

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\* W. von Ignatowski, "Einige allgemeine Bemerkungen über das Relativitätsprinzip," *Physikalische Zeitschrift* 11 (1910): 972–976, with a fuller treatment in *Archiv der Mathematik und Physik* the following year; the derivation was tightened by P. Frank and H. Rothe, "Über die Transformation der Raum-Zeitkoordinaten von ruhenden auf bewegte Systeme," *Annalen der Physik* 34 (1911): 825–855.

† A. A. Robb, *A Theory of Time and Space* (Cambridge: Cambridge University Press, 1914). Robb called his primitive relation "conical order" in the 1921 sequel. The book's reception is its own small tragedy; it was admired, cited, and almost never read through.

‡ H. Reichenbach, *Axiomatik der relativistischen Raum-Zeit-Lehre* (Braunschweig: Vieweg, 1924); A. D. Alexandrov, "A Contribution to Chronogeometry," *Canadian Journal of Mathematics* 19 (1967): 1119–1128, reporting results first published in Russian in 1950; E. C. Zeeman, "Causality Implies the Lorentz Group," *Journal of Mathematical Physics* 5 (1964): 490–493. Alexandrov and Zeeman arrived independently; the priority belongs to Alexandrov, the title to Zeeman.

**1958 — Salecker and Wigner.** A clock is a quantum system, and quantum systems spread. Give the clock too little mass and the spreading swallows it: it cannot keep time. Keeping good time takes mass, and trusting the reading takes structure. They proved a working clock cannot be a point.<sup>19</sup>§ They proved the observer must have size. The field read it as a limit on laboratory clocks, and moved on.§

**1964; 1972 — Marzke and Wheeler; Ehlers, Pirani, and Schild.** Two attempts to pay Einstein's debt directly. In 1964 Marzke and Wheeler built the "geometrodynamical standard meter": rods and clocks constructed from light rays and free particles, so the theory measures itself with its own contents. In 1972 Ehlers, Pirani, and Schild completed the program: start from how light propagates and how bodies fall, and the geometry of spacetime follows.<sup>20</sup>¶ This pays half the debt. The rods and clocks of 1905 are no longer stipulated equipment; they are built from physics. But check what they are built from. A light ray is a null geodesic: a curve. A free particle is a timelike geodesic: a curve. Every instrument is assembled from idealized point-paths. The 1949 question, what must the observer itself be, was not answered here. It was not asked. Salecker and Wigner were completely ignored. Einstein's lament: left open.

**1976; 1984 — Lévy-Leblond; Mermin.** The von Ignatowski route distilled into four assumptions and a famous pedagogical paper; relativity taught without light.<sup>21</sup>¶

I met these men only after my own result was published; von Ignatowski entered my bibliography before he entered my knowledge. Reading him at last did nothing to me. A footnote. A similar reduction, and he still assumed the point. They had shaved the theory to what felt to them the bare minimum, but left the idealized point. § VI offers the only explanation I find credible: It probably was asked on scratch paper by someone fluent in these axioms but with no use in hand; judged a curiosity and thrown away.

## V. The Paradoxes That Kept Knocking, 1959–1979

*"I conclude that the problem of the reality and the determinateness of future events is now solved. Moreover, it is solved by physics and not by philosophy."* — Putnam, 1967

The finite body issue was suppressed in 1911. It came up anyway, over the next seventy years. Five times, each answered as a puzzle, no common solution.

**1959 — Terrell; Penrose.** Fifty-four years after the founding paper, someone finally asked what a moving body actually looks like. The answer surprised everyone: the Lorentz contraction is invisible. A photograph of a fast sphere shows a rotation, never a flattening.<sup>22</sup>\* The most famous effect in the theory hides from every finite viewer. The result was admired, taught, and filed as an optical curiosity.

The shadow was doing the physics. I had not found it yet.

**1959 — Dewan and Beran.** Two rockets, identical accelerations, a thread stretched between them. Does it break? The kinematics alone cannot say; the question has no answer until the thread is treated as a body with parts. Bell revived the puzzle at CERN in 1976, put it to the theory division, and reported a clear

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§ H. Salecker and E. P. Wigner, "Quantum Limitations of the Measurement of Space-Time Distances," *Physical Review* 109 (1958): 571–577. The result's next life came three decades later, as a limit in quantum-gravity phenomenology: G. Amelino-Camelia, "On the Salecker–Wigner Limit and the Use of Interferometers in Space-Time-Foam Studies," *Physics Letters B* 477 (2000): 1–7.

¶ R. F. Marzke and J. A. Wheeler, "Gravitation as Geometry — I: The Geometry of Space-Time and the Geometrodynamical Standard Meter," in H.-Y. Chiu and W. F. Hoffmann, eds., *Gravitation and Relativity* (New York: W. A. Benjamin, 1964); J. Ehlers, F. A. E. Pirani, and A. Schild, "The Geometry of Free Fall and Light Propagation," in L. O'Raifeartaigh, ed., *General Relativity: Papers in Honour of J. L. Synge* (Oxford: Clarendon Press, 1972).

¶ J.-M. Lévy-Leblond, "One More Derivation of the Lorentz Transformation," *American Journal of Physics* 44 (1976): 271–277; N. D. Mermin, "Relativity Without Light," *American Journal of Physics* 52 (1984): 119–124.

\* J. Terrell, "Invisibility of the Lorentz Contraction," *Physical Review* 116 (1959): 1041–1045; R. Penrose, "The Apparent Shape of a Relativistically Moving Sphere," *Proceedings of the Cambridge Philosophical Society* 55 (1959): 137–139. Independent, same year, half a century late. The reception, in the field's own magazine: V. F. Weisskopf, "The Visual Appearance of Rapidly Moving Objects," *Physics Today* 13, no. 9 (1960): 24–27 — Terrell had challenged an old prejudice held by practically all physicists.

consensus for the wrong answer.<sup>23†</sup> An extended object, the moment it enters relativity, forces questions a point kinematics can't answer.

The shadow could answer them. I was circling it without the math.

**1962 — Hartman.** Tunneling: a particle crosses a barrier it has no energy to climb. Hartman computed the time the crossing takes and found it stops growing with the barrier's width; make the wall thicker and the transit time stays put, which, read as a speed, is faster than light.<sup>24‡</sup> The standard telling: the probability field leaks through. It was filed as a puzzle of waves.

Tunneling was my biggest push. Not the data, the telling. Walking through walls is not an explanation; it is an admission that matter and boundaries were not understood. I learned much later that Einstein had refused the dice on the same instinct.<sup>25§</sup>

**1966; 1967 — Rietdijk; Putnam.** The still frame took its knock too. From nothing but the relativity of simultaneity, Rietdijk argued determinism outright; Putnam declared the open future an illusion; both had derived the same underlying fact: no invariant "now" exists.<sup>26¶</sup> The theorem was sound. The field filed it under philosophy and went on computing on constant-time slices.

Putnam declared it solved the year I was born. Two decades later I spent much of my time trying to picture a still frame of the missing space, and could not. And math could not see it either. Only the propagation, the hiding, could be seen or properly encoded by the math: the action, step by step, of hiding space. I did not know their theorem but inherited their problem.

**1976; 1979 — Unruh; DeWitt.** When acceleration met the quantum, the observer finally had to be modeled, and was modeled as a point. Unruh's detector is a two-level system on a worldline, warming in its own acceleration.<sup>27¶</sup> DeWitt's monopole detector simplified it further. The choice was deliberate and sensible: a detector with size feels different accelerations at its two ends, so size was assumed away. For one decade the observer was almost physics. The blind spot held.

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† E. Dewan and M. Beran, "Note on Stress Effects due to Relativistic Contraction," *American Journal of Physics* 27 (1959): 517–518; J. S. Bell, "How to Teach Special Relativity" (1976), reprinted in *Speakable and Unspeakable in Quantum Mechanics* (Cambridge: Cambridge University Press, 1987). Bell's own account: he put the problem to the CERN theory division, and a clear consensus emerged that the thread would not break. It breaks. Dewan and Beran had it right in 1959, and were doubted then too. The arc has since been drawn in one sentence: "The controversy about the reality of the length contraction that was settled in the 1910s on the pages of the *Physikalische Zeitschrift* erupted again in the 1970s at the tables of the CERN canteen." *European Journal for Philosophy of Science* (2023).

‡ T. E. Hartman, "Tunneling of a Wave Packet," *Journal of Applied Physics* 33 (1962): 3427–3433. The saturation is now called the Hartman effect. Six decades of dispute over what, if anything, crosses faster than light; the dispute is about the telling, not the data: H. G. Winful, "Tunneling Time, the Hartman Effect, and Superluminality," *Physics Reports* 436 (2006): 1–69.

§ A. Einstein to M. Born, 4 December 1926, in *The Born–Einstein Letters* (London: Macmillan, 1971). In the original: "Die Theorie liefert viel, aber dem Geheimnis des Alten bringt sie uns kaum näher. Jedenfalls bin ich überzeugt, daß der nicht würfelt." The theory delivers much, but brings us hardly nearer to the secret of the Old One; in any case, he does not dice. The letter answers Born's probability rule, then months old: M. Born, "Zur Quantenmechanik der Stoßvorgänge," *Zeitschrift für Physik* 37 (1926): 863–867. The probability that leaks through Hartman's wall above is Born's. The dispute never closed; it became an experiment instead: J. S. Bell, "On the Einstein Podolsky Rosen Paradox," *Physics* 1 (1964): 195–200; A. Aspect, J. Dalibard, and G. Roger, *Physical Review Letters* 49 (1982): 1804–1807. The experiments ruled; the telling is argued still.

¶ C. W. Rietdijk, "A Rigorous Proof of Determinism Derived from the Special Theory of Relativity," *Philosophy of Science* 33 (1966): 341–344; H. Putnam, "Time and Physical Geometry," *Journal of Philosophy* 64 (1967): 240–247. The argument never died; Penrose, met above photographing spheres, rebuilt it in 1989 as the Andromeda paradox: R. Penrose, *The Emperor's New Mind* (Oxford: Oxford University Press, 1989).

¶¶ W. G. Unruh, "Notes on Black-Hole Evaporation," *Physical Review D* 14 (1976): 870–892; B. S. DeWitt, "Quantum Gravity: The New Synthesis," in S. W. Hawking and W. Israel, eds., *General Relativity: An Einstein Centenary Survey* (Cambridge: Cambridge

University Press, 1979). The detector's temperature:  $T = \frac{\hbar a}{2\pi c k_B}$ . Planck's constant beside the acceleration; even the point detector's thermometer runs on  $\hbar$ .

## VI. The Mathematics Arrives, 1972–1999

"Mathematics is a part of physics. Physics is an experimental science, a part of natural science. Mathematics is the part of physics where experiments are cheap." — Arnold, 1997

While the foundations closed tighter, mathematics was quietly building everything the payment would need. Nobody I could find told either side.

**1972 — Thom.** *Stabilité structurelle et morphogénèse* classified the ways a smooth family of solutions can fold and break: a short list of elementary catastrophes, universal and unavoidable.<sup>28\*</sup> The intended applications were embryology and language, almost anything but light.

**1980 — Berry and Upstill.** A caustic is where a family of light rays folds and crosses itself. Ray theory predicts infinite brightness there: a singularity. Berry and Upstill catalogued what light actually does at these places. It does not blow up. It forms fixed patterns, the same ones every time; at a simple fold they are Airy's fringes, known since 1838.<sup>29†</sup> In gravity a singularity is where the laws fail. Here is where the light is brightest and the mathematics works best. And the same integrals sit inside quantum mechanics, with  $\hbar$  in the exponent where optics carries the wavelength.<sup>‡</sup> Quantum mechanics rests on Schrödinger's equation,  $i\hbar \frac{d}{dt}|\psi\rangle = \hat{H}|\psi\rangle$ . The equation does not derive its wave; it assumes one, of the form  $e^{iS/\hbar}$ . Working from that assumed wave down to the fold was standard practice with a name: semiclassical mechanics. The other direction had no name.

**1984 — Green and Schwarz.** An anomaly cancelled, and the superstring revolution began.<sup>30‡</sup> Within a few years it was arguably the largest employer of theoretical talent in the world. The promise was a final theory; price was ten dimensions and mathematics generations deep. The mathematics it made was real.

**1985 — Arnold; Gusein-Zade; Varchenko.** The complete apparatus: two volumes, probably over 300 theorems and lemmas, real mathematics for real folds in real space.<sup>31§</sup> I used just a few.

Arnold had already shown, mathematically, what a thing with structure does in real space when it meets a caustic. The missing plank, the one I was looking for and no one else saw: for structure traversing at the speed of light (light modeled as Einstein wanted), where the travel and the response are the same speed, that result is  $e^{iS/\hbar}$ . No new mathematics was needed; Arnold had provided the definitive collection, and it stands today. Only a premise nobody carried into this mathematics: that light has structure.

By 1985, everything was on the shelf. Across the century, fluent hands must have tried pieces of it. Someone modeled light with structure, and someone watched closed loops give back gravity's far field but discarded them as **reproductions, nothing more**. Someone, maybe many, ran the finite observer against § IV's axioms and got its odd persistence condition: a curiosity, nothing more. All **thrown away**, and **reasonably so**.

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\* R. Thom, *Stabilité structurelle et morphogénèse* (Reading: W. A. Benjamin, 1972); English translation, *Structural Stability and Morphogenesis* (1975).

† M. V. Berry and C. Upstill, "Catastrophe Optics: Morphologies of Caustics and Their Diffraction Patterns," *Progress in Optics* 18 (1980): 257–346. The two-sided correspondence is Berry's own territory: M. V. Berry and K. E. Mount, "Semiclassical Approximations in Wave Mechanics," *Reports on Progress in Physics* 35 (1972): 315–397. That the wave is assumed in the form  $e^{iS/\hbar}$  is the founding move of the path-integral formulation itself: P. A. M. Dirac, "The Lagrangian in Quantum Mechanics" (1933); R.P. Feynman, "Space-Time Approach to Non-Relativistic Quantum Mechanics," *Reviews of Modern Physics* 20 (1948): 367–387.

‡ M. B. Green and J. H. Schwarz, "Anomaly Cancellations in Supersymmetric D=10 Gauge Theory and Superstring Theory," *Physics Letters B* 149 (1984): 117–122.

§ V. I. Arnold, S. M. Gusein-Zade, and A. N. Varchenko, *Singularities of Differentiable Maps*, 2 vols. (Boston: Birkhäuser, 1985–1988).

Without the generator for  $e^{iS/\hbar}$ , none of that mathematics connects to anything. Scribbles, musings, results without a use. I kept mine only for the promise riding on it.

**1986 — Glashow and Ginsparg.** The question came from inside the house, in a title: "Desperately Seeking Superstrings?"<sup>32</sup>|| Glashow held a Nobel Prize; this was not an outsider's complaint. The question was whether string theory was physics at all: no prediction made, no experiment proposed, none possible at reachable energies. No experiment ever answered the question. Hiring did.

1988 to 1992 were my conception years. One room was adding dimensions; another was finishing the geometry of folds. I was removing an assumption, and knew of neither.

**1988 — Hawking.** A Brief History of Time.<sup>33</sup>¶ His non-uniform Big Bang. From the father of the Big Bang. The Big Bang that had tied me up in logic. The theorem stuck me. The book unstuck me. I read it after everyone read it, around 1994, and it broke the uniformity dead-end I had been carrying. My missing-space model needed a non-uniform universe to begin in. Ten million people bought that book. I may have been the only one it unstuck in quite that way.

**1988 — Feynman.** In the year he died, on strings: "I don't like that they're not calculating anything." Not one number to check against the world, and disagreements explained away instead of tested.<sup>34</sup>\*\* The objection was published in a book celebrating the theory.

**1989 — Mermin.** The doctrine got its name. In Physics Today, Mermin summed up the Copenhagen attitude in three words: "shut up and calculate."<sup>35</sup>†† The phrase is usually credited to Feynman; it is Mermin's, and he offered it as reproach. The field kept it as advice. The same Mermin had rebuilt relativity without light. The sharpest distiller of the audit tradition also named the doctrine of not-auditing. Four years earlier the apparatus in Moscow was finished, printed, and shelved.

**1990 — Bell.** "Against 'measurement,'" published in physics' house journal the year he died: the demand that the load-bearing words be defined or retired: measurement, observer, apparatus.<sup>36</sup>‡‡ He reached back for Schrödinger's cat, built in 1935 as a protest against the theory's account of observation and worn ever since as its mascot. Fifty-five years, and the protest was still being read backwards.

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|| S. L. Glashow and P. Ginsparg, "Desperately Seeking Superstrings?," Physics Today 39, no. 5 (1986): 7–9. Glashow shared the 1979 Nobel Prize for electroweak unification. From the article: superstring study risks becoming an activity "conducted at schools of divinity by future equivalents of medieval theologians." The view from inside was franker: Gross, interviewed in the same volume that carried Feynman's dissent, said the attraction was that there was no other game in town. On what the incentives then did to two decades of careers: L. Smolin, The Trouble with Physics (Boston: Houghton Mifflin, 2006); P. Woit, Not Even Wrong (New York: Basic Books, 2006).

¶ S. W. Hawking, A Brief History of Time (New York: Bantam, 1988). I know. The father of the Big Bang is Lemaître, the priest who gave the expanding universe its equations in 1927 and the "primeval atom" in 1931, standing on Friedmann's 1922 mathematics: G. Lemaître, Annales de la Société Scientifique de Bruxelles A47 (1927): 49–59, and Nature 127 (1931): 706; A. Friedmann, "Über die Krümmung des Raumes," Zeitschrift für Physik 10 (1922): 377–386. Hawking's title is different: with Penrose he proved the beginning unavoidable, the singularity guaranteed by theorem: S. W. Hawking and R. Penrose, "The Singularities of Gravitational Collapse and Cosmology," Proceedings of the Royal Society A 314 (1970): 529–548. It says nothing about texture. The book's argument, that structure requires a non-uniform start, came in 1988, ahead of the evidence; COBE measured the ripples in 1992, two years before the book reached me. But that is not how the book arrived for me in 1994. It arrived as the Big Bang's own book, from its owner.

\*\* R. P. Feynman, interviewed in P. C. W. Davies and J. Brown, eds., Superstrings: A Theory of Everything? (Cambridge: Cambridge University Press, 1988). The complaint in full: they do not calculate, they do not check, and whatever disagrees with experiment gets "a fix-up to say, 'Well, it still might be true.'" He died that February; the book of celebrations carried the dissent to press.

†† N. D. Mermin, "What's Wrong with This Pillow?," Physics Today 42, no. 4 (1989): 9. The sentence in full: "If I were forced to sum up in one sentence what the Copenhagen interpretation says to me, it would be 'Shut up and calculate!'" His next words: "But I won't shut up." The phrase is so often credited to Feynman that Mermin published the correction himself: "Could Feynman Have Said This?," Physics Today 57, no. 5 (2004): 10.

‡‡ J. S. Bell, "Against 'Measurement,'" Physics World 3, no. 8 (1990): 33–40. The cat: E. Schrödinger, "Die gegenwärtige Situation in der Quantenmechanik," Naturwissenschaften 23 (1935): 807–812, 823–828, 844–849; published in three installments. His register was satire: "ganz burleske Fälle," rendered in the standard translation as "quite ridiculous cases": J. D. Trimmer, "The Present Situation in Quantum Mechanics," Proceedings of the American Philosophical Society 124 (1980): 323–338.

None of this was hidden. Heisenberg had written that the classical concepts were known to be inexact and that "we cannot and should not try to improve them."<sup>37</sup> §§ Dirac called the reigning mathematics not sensible. Feynman told his own readers the procedure was "hocus-pocus." Kuhn had named the mechanism in 1961, in a title: the function of dogma in scientific research. They knew, they said so in print, and the field kept computing. I make no charge the record does not make itself.

**1999 — Barbour.** The End of Time: the opposite bet, published the same year the wave manual was finished. Reality as stills, motion as the illusion.<sup>38</sup> || I had been on that road once; the still frame was the first thing the work made me surrender. His question was time. Mine was what causes curvature: what mass is, and that needed time.

**1999 — Kravtsov and Orlov.** Caustics, Catastrophes and Wave Fields: the working manual. Waves through real media, the uniform methods, the thin-layer treatment.<sup>39</sup> ¶¶ The tools were now complete, published, and shelved. They waited twenty-six years.

For the field, this mathematics arrived over three decades. For me it arrived in 2024 and 2025, as I was compiling four decades of notes. A dead laptop held the old work: Minkowski's forms, where in the linear regime I had shown the circling paths produced gravity and resolved exactly to Einstein's weak-field equations. I searched for a way to fix it, gave up, and rebuilt the work from scratch. Then I took another stab at the missing space: what it looks like to model space with absence instead of a mass-energy tensor.

Dozens of forms came and went. Even then the point rode along: everything still referenced to a center, the oldest habit in physics. And in the math the volume would cancel in every final calculation.

What the cancellation meant, that waited for another six months: a finite observer has one invariant, and the invariant is what identity is. I was just happy to derive the classic frameworks. Alone, that result was a scribble. A persistence condition with no use in hand. The use is what the mathematics enabled, making my theory's assumptions mathematical consequences all along.

Transformations between observers are linear and reciprocal; there is one invariant speed; a finite invariant observer. Those three alone, which all classic frameworks carry, but without the point assumption, produced my original postulates.<sup>40</sup> \*\*\*

I should have done all of it decades sooner. I begged the wrong people, asked the wrong questions, and never took on the quantum directly until the very end though it was the heart of the issue. The blind spot does not check credentials. My apologies so tendered, in full.

When the mathematics was done, I bought the books. Thom, Arnold, Kravtsov and Orlov. My solution was already inside them, worked out decades earlier, for lenses. I had never opened them. I found the pieces I needed online. The shelf had been waiting the whole time.

Einstein deferred his debt to a later stage of the theory.

This is the later stage.

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§§ Four confessions, one note. W. Heisenberg, *Physics and Philosophy* (New York: Harper, 1958). P. A. M. Dirac, *Directions in Physics* (New York: Wiley, 1978), on renormalization: "this is just not sensible mathematics." R. P. Feynman, *QED: The Strange Theory of Light and Matter* (Princeton: Princeton University Press, 1985), "hocus-pocus." T. S. Kuhn, "The Function of Dogma in Scientific Research," in A. C. Crombie, ed., *Scientific Change* (London: Heinemann, 1963), 347–369; delivered Oxford, 1961. Kuhn on the mechanism itself: "Preconception and resistance seem the rule rather than the exception in mature scientific development... they characterize the very best and most creative research as well as the more routine." And Einstein, to Besso, 1951: "Die ganzen fünfzig Jahre bewußter Grübeleien haben mich der Antwort auf die Frage 'Was sind Lichtquanten?' nicht näher gebracht." Fifty years of conscious brooding, no nearer to what light quanta are.

||| J. Barbour, *The End of Time* (London: Weidenfeld & Nicolson, 1999).

¶¶ Yu. A. Kravtsov and Yu. I. Orlov, *Caustics, Catastrophes and Wave Fields*, 2nd ed. (Berlin: Springer, 1999).

\*\*\* The three are the Lemma's assumptions, A1 through A3, stated there formally; the third gives the finite observer its invariance.

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